$\omega - \phi$ MIXING AND WEAK ANNIHILATION IN D_s DECAYS

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The mixing between nonstrange and strange quark wavefunctions in the ω and ϕ mesons leads to a small predicted branching ratio $\mathcal{B}(D_s^+ \to \omega e^+ \nu_e) = \mathcal{O}(10^{-4})(\delta/3.34°)^2$, where δ is the mixing angle. The value $\delta = -3.34°$ is obtained in a mass-independent analysis, while a mass-dependent analysis gives $\delta = -0.45°$ at $m(\omega)$ and $-4.64°$ at $m(\phi)$. Measurement of this branching ratio thus can tell whether the decay is dominated by ϕ–ω mixing, or additional nonperturbative processes commonly known as "weak annihilation" (WA) contribute. The role of WA in the decay $D_s^+ \to \omega \pi^+$ and its possible use in estimating WA effects in $D_s^+ \to \omega e^+ \nu_e$ are also discussed. Assuming that the dynamics of WA in $D_s^+ \to \omega \pi^+$ is similar in $D_s^+ \to \omega e^+ \nu_e$ we estimate $\mathcal{B}(D_s^+ \to \omega e^+ \nu_e) = (1.3 \pm 0.5) \times 10^{-3}$.

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I. INTRODUCTION

The CLEO Collaboration has completed a study of $e^+ e^-$ production of charmed mesons near threshold, including a sample of about 600 pb^{-1} at $\sqrt{s} = 4.17$ GeV [1] where $D_s^+ D_s^{*-} + D_s^- D_s^{*+}$ pairs are produced with a cross section approaching 1 nb. This permits the study of rare D_s meson decays.

A prominent semileptonic decay of D_s mesons is the process $D_s^+ \to \phi e^+ \nu_e$, with branching ratio [2]

$$\mathcal{B}(D_s^+ \to \phi e^+ \nu_e) = (2.36 \pm 0.26)\% . \tag{1}$$

At the quark level, this is represented by the process $c \to s e^+ \nu_e$, with the final s and the spectator $\bar{s}$ forming a ϕ. One anticipated contribution to the process $D_s^+ \to \omega e^+ \nu_e$, on the other hand, is expected to involve the small $s\bar{s}$ admixture in the ω wave function, and hence to be highly suppressed by the Okubo-Iizuka-Zweig (OZI) [3] rule. An additional process,

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commonly known as "weak annihilation" (WA) [4], would involve nonperturbative pre-radiation of an ω meson by the $c\bar{s}$ system, followed by the annihilation process $c\bar{s} \to e^+\nu_e$. A similar WA process can account for at most a few percent of B meson semileptonic decays to charmless final states [5, 6]. The effects of WA in semileptonic D_s decays are expected to be considerably larger than in semileptonic charmless B decays. We shall estimate their contribution to $D_s^+ \to \omega e^+ \nu_e$ and discuss their characteristic kinematic signatures.

One should distinguish between two types of WA often discussed in the literature. In D_s decays, if the c and $\bar{s}$ annihilate weakly into $u\bar{d}$, and the $u\bar{d}$ state then materializes into a non-strange final states such as $\pi^+\pi^+\pi^-$, there is, in principle, no OZI suppression, although helicity conservation arguments for the light u and $\bar{d}$ quarks lead one to expect a suppression of the amplitude unless at least two gluons also pass from the initial to the final state [7]. Such WA processes (when exchange amplitudes, which are also in principle subject to helicity suppression, are included) are likely to be a major source of charmed particle lifetime differences. On the other hand, we are considering a form of WA which involves OZI-suppressed nonperturbative pre-radiation of an isoscalar system such as an ω meson, e.g., in $D_s \to \omega(D_s^*)_{\text{virtual}} \to \omega\ell\nu$. It is this type of WA whose contribution to B charmless semileptonic decays can affect the extraction of $|V_{ub}|$ from such processes [4].

A recent discussion of the effects of ω–ϕ mixing in B meson decays may be found in Ref. [8]. The physical states may be represented in terms of ideally mixed states $\omega^I \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, $\phi^I \equiv s\bar{s}$ by

$$\begin{pmatrix} \omega \\ \phi \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} \omega^I \\ \phi^I \end{pmatrix} \qquad (2)$$

In one mass-independent analysis [9] a mixing angle $\delta = -(3.34 \pm 0.17)^\circ$ was obtained, while allowing for energy dependence [10] one finds δ varying from -0.45° at $m(\omega)$ to -4.64° at $m(\phi)$. A measurement of $\mathcal{B}(D_s^+ \to \omega e^+\nu_e)$ can help distinguish between these predictions and uncover any new effects beyond those associated with ϕ–ω mixing. In this article we predict the relation between $\mathcal{B}(D_s^+ \to \omega e^+\nu_e)$ and the mixing angle, and note how further data on $D_s^+ \to \phi e^+\nu_e$ can sharpen the prediction, potentially providing also information on a contribution of weak annihilation. We also discuss the role of weak annihilation in the hadronic two-body decay $D_s^+ \to \omega\pi^+$.

The treatment of D_s decays benefits from a heavy-quark symmetry framework described in Section II. The approach is applied here to the semileptonic decays $D_s^+ \to (\omega, \phi, \eta, \eta')\ell^+\nu_\ell$, and to the two-body hadronic decays $D_s^+ \to (\omega, \phi, \eta, \eta')\pi^+$ in Section III. We discuss a possible connection between WA in $D_s^+ \to \omega\pi^+$ and $D_s^+ \to \omega e^+\nu_e$ in Section IV, and conclude in Section V.

II. SEMILEPTONIC D_s DECAYS

A. Kinematic and form factor effects

We begin by comparing kinematic and form factor effects on the decays $D_s \to (\omega_s, \phi)\ell\nu$, where ω_s is a ficitious particle with the mass of ω and the pure-strange-quark content of an ideally mixed ϕ_I. The phase space for decay of a particle of mass M to three final-state

2

particles, one of which has mass m and the other two of which are massless, is reduced with respect to that for three massless final particles by a factor $g(x) \equiv 1 - x^2 + 2x \ln x$, where $x \equiv (m/M)^2$. Applying this to the decays $D_s \to (\omega, \phi)\ell\nu$, we find $x = (0.158, 0.268)$ and $g(x) = (0.392, 0.222)$ for (ω, ϕ). The ratio of these two values is 1.76. This kinematic factor is the one appropriate for a flat Dalitz plot. For the decay of a fermion of mass M to another of mass m and two massless fermions with $(V - A) \times (V - A)$ coupling, as in $\tau^- \to \mu^- \nu_\tau \bar{\mu}_\nu$, the appropriate function would instead be $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$, equal to $(0.3195, 0.1395)$ for (ω, ϕ). The ratio of these last two values is 2.3.

Form factors can affect the ratio $R_s \equiv \Gamma(D_s \to \omega_s \ell\nu)/\Gamma(D_s \to \phi_I \ell\nu))$. In the heavy-quark formalism of Refs. [11], as applied in Ref. [12], a single form factor governs all the helicity amplitudes for the decays of a pseudoscalar meson to a vector or pseudoscalar meson and a lepton pair. We employ this formalism primarily to illustrate the possible variations from the flat-Dalitz-plot value of $R_s = 1.76$. We find a range $1.2 \leq R_s \leq 2.4$ in various applications of the symmetry, and shall consider these rather conservative bounds in estimating the rate for $D_s \to \omega\ell\nu$ due to ω–ϕ mixing.

To the extent that the strange quark's effective mass in $D_s \to \phi\ell\nu$ can be regarded as 0.5 GeV/c^2 (its "constituent-quark" mass), the heavy-quark limit discussed in Ref. [12] begins to have some validity. Its limitations for the related processes $D \to (\overline{K}^*, \overline{K})\ell\nu$ were discussed extensively in Ref. [13]. Primary among these limitations is the importance of $1/m_s$ corrections in reducing the predicted branching ratio for $D \to \overline{K}^*\ell\nu$ by about a factor of two while affecting the predicted branching ratio for $D \to \overline{K}\ell\nu$ much less. We shall see that similar effects are called for when comparing $D_s \to \phi\ell\nu$ with $D_s \to (\eta, \eta')\ell\nu$. A form factor parameter which describes $D_s \to (\eta, \eta')\ell\nu$ adequately will be seen to predict a branching ratio for $D_s \to \phi\ell\nu$ about a factor of 2 above experiment. As D_s semileptonic decays are related to D semileptonic decays by replacement of a nonstrange by a strange spectator quark, we expect the pattern of $1/m_s$ corrections in the former to be the same as in the latter, for which a satisfactory description was obtained [13].

The universal form factor $\xi(w^2)$ is a function of the invariant square of the universal velocity transfer $w = v - v'$, with $v = p_{D_s}/M_{D_s}$ and $v' = p_V/m_V$. In terms of the invariant square $q^2 = m_{e\nu}^2$ of the 4-momentum transfered to the lepton pair, one has

$$w^2 = \frac{q^2 - q_{\max}^2}{M_{D_s} m_V} = \frac{q^2 - (M_{D_s} - m_V)^2}{M_{D_s} m_V} \ . \tag{3}$$

The form factor $\xi(w^2)$ is normalized to unity at $w^2 = 0$ aside from a QCD enhancement factor E [14]:

$$\xi(w^2) = \frac{E}{1 - w^2/w_0^2} \ , \quad E \equiv \left[\frac{\alpha_s(M_{D_s}^2)}{\alpha_s(m_V^2)} \right]^{-6/(33-2n_f)} \ . \tag{4}$$

We take the number of flavors n_f equal to three. For our purposes it is sufficient to estimate $\alpha_s(M_{D_s}^2) = 0.3$, $\alpha_s(m_V^2) = 0.4$, so the QCD enhancement factor is $E = (3/4)^{-2/9} = 1.066$.

The differential decay rates with respect to the dimensionless parameter $y = q^2/M_{D_s}^2$ for pseudoscalar mesons P and for transversely and longitudinally polarized vector mesons $V_{T,L}$ are then [12]

$$\frac{d\Gamma_p}{dy} = \frac{\Gamma_0 \lambda^{1/2}(1, \zeta, y) f_p(y)}{(1 - w^2/w_0^2)^2} \ , \tag{5}$$

3

$$\Gamma_0 \equiv \frac{(G_F V_{cs} E)^2 M_{D_s}^5}{192\pi^3} = 7.28 \times 10^{-13} \text{ GeV} \tag{6}$$

for $V_{cs} = 0.974$ and $M_{D_s} = 1968.5$ MeV/c^2. Here $\lambda(a,b,c) \equiv a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, $\zeta \equiv m_V^2/M_{D_s}^2$, while

$$f_p(y) \equiv \begin{cases} (1+\sqrt{\zeta})^2 \lambda(1,\zeta,y)/4\sqrt{\zeta} & p = Pe\nu \,, \\ y[(1+\sqrt{\zeta})^2 - y](1+\zeta-y)/\sqrt{\zeta} & p = V_T e\nu \,, \\ (1-\sqrt{\zeta})^2[(1+\sqrt{\zeta})^2 - y]^2/4\sqrt{\zeta} & p = V_L e\nu \,. \end{cases} \tag{7}$$

For $\tau_{D_s} = 0.500 \pm 0.007$ ps [2], the corresponding differential branching ratios with respect to y are then $dB_p/dy = 0.55\lambda^{1/2}(1,\zeta,y)f_p(y)/(1 - w^2/w_0^2)^2$.

In Ref. [13], a parametrization for the universal monopole form factor was adopted with $w_0 = \sqrt{2}/\rho$, $\rho = 1.00 \pm 0.15$. This is equivalent to $1.23 \leq w_0 \leq 1.66$, a range which will be of particular interest to us.

B. Polarization and branching ratios

For the decays $D_s \to (\omega_s, \phi)\ell\nu$, the sums $d\mathcal{B}/dy \equiv d\mathcal{B}_T/dy + d\mathcal{B}_L/dy$ are plotted for several values of w_0 in Fig. 1. The value $w_0 = 0.5$ is slightly below the lowest value providing a fit to the total branching ratio $\mathcal{B}(D_s^+ \to \phi e^+ \nu_e)$; the value $w_0 = 1.7$ is slightly above the highest value considered for the universal form factor in Ref. [13]; the value at $w_0 = \infty$ corresponds to no form factor damping.

The branching ratio $\mathcal{B}(D_s^+ \to \phi e^+ \nu_e) = \int_0^{y_{\max}} dy (d\mathcal{B}/dy)$, the ratio $\mathcal{B}_L(\phi)/\mathcal{B}_T(\phi)$ of longitudinal to transverse decay rates, and the ratio $R_s = \mathcal{B}(\omega_s)/\mathcal{B}(\phi)$ are plotted in Fig. 2. Values of w_0 between about 0.53 and 0.63 yield satisfactory values of $\mathcal{B}(D_s^+ \to \phi e^+ \nu_e)$. For $w_0 = (0.5, 1.7, 2.2, \infty)$ the ratios R_s are $(1.24, 2.07, 2.18, 2.41)$. We thus consider $1.2 \leq R_s < 2.4$ as a conservative range. Although lower values are associated with values of w_0 giving a better fit to $\mathcal{B}(D_s^+ \to \phi e^+ \nu_e)$, consideration of the semileptonic decays $D_s \to (\eta, \eta')\ell\nu$ and the hadronic decay $D_s^+ \to \phi\pi^+$ favors the higher ratio.

The ratio of longitudinal to transverse polarization in $D_s^+ \to \phi e^+ \nu_e$ is predicted to range between 0.92 and 1.22 for $0.5 \leq w_0 \leq 2.5$, whereas the Particle Data Group average is quoted as 0.72 ± 0.18 [2], based on the individual measurements [15, 16, 17] quoted in Table I. (Form factor ratios have been measured recently more precisely by the E791 [18] and FOCUS [19] Collaborations at Fermilab and by the BaBar Collaboration at SLAC [20], but their ratios $\mathcal{B}_L(\phi)/\mathcal{B}_T(\phi)$ were not directly quoted.)

Neglecting for a moment weak annihilation, the ratio $R \equiv \mathcal{B}(D_s^+ \to \omega e^+ \nu_e)/\mathcal{B}(D_s^+ \to \phi e^+ \nu_e)$ is governed by several effects: (1) a phase space correction, (2) a difference between form factors, and (3) the $\omega - \phi$ mixing angle. We have estimated that the product of the first two gives a range $1.2 \leq R_s \leq 2.4$ for ω_s and ϕ_I composed entirely of $s\bar{s}$. The mixing angle then implies $R = R_s \tan^2 \delta$, where $\tan^2 \delta = 3.41 \times 10^{-3}$ for $\delta = -3.34°$. We then find $R = (4.1 - 8.2)(\delta/3.34°)^2 \times 10^{-3}$, implying [when we take also $\pm 1\sigma$ errors on $\mathcal{B}(D_s^+ \to \phi e^+ \nu_e)$] that

$$\mathcal{B}(D_s^+ \to \omega e^+ \nu_e) = (0.9 - 2.1) \times 10^{-4} \left(\frac{\delta}{3.34°}\right)^2 \,. \tag{8}$$

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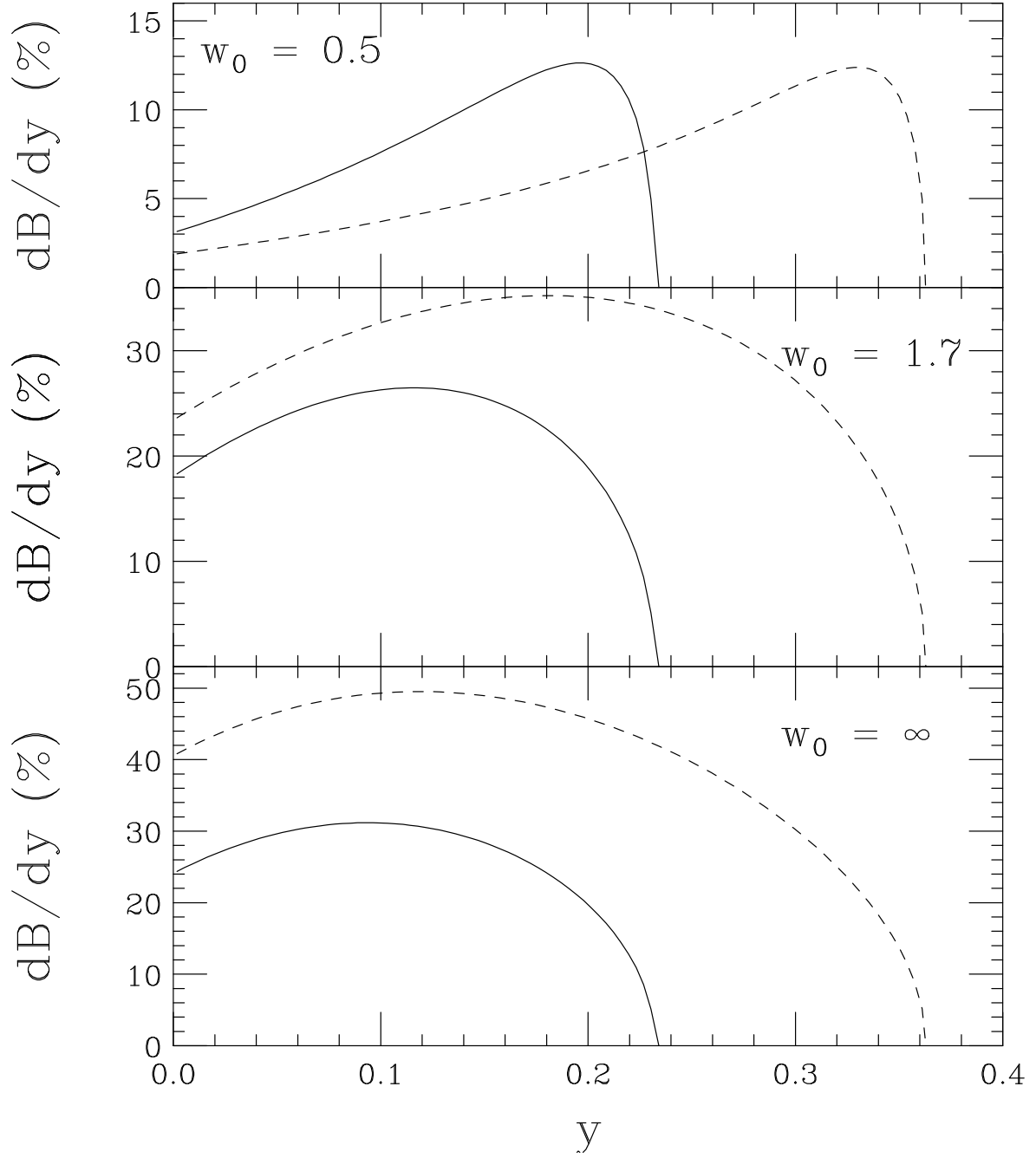
Figure 1: Values of dB/dy (in percent) for the processes $D_s^+ \to \phi e^+ \nu_e$ (solid curves) and $D_s^+ \to \omega_s e^+ \nu_e$ (dashed curves, where ω_s denotes a pure $s\bar{s}$ state with the mass of ω), for several values of the form factor parameter w_0. Top: $w_0 = 0.5$; middle: $w_0 = 1.7$; bottom: $w_0 = \infty$.
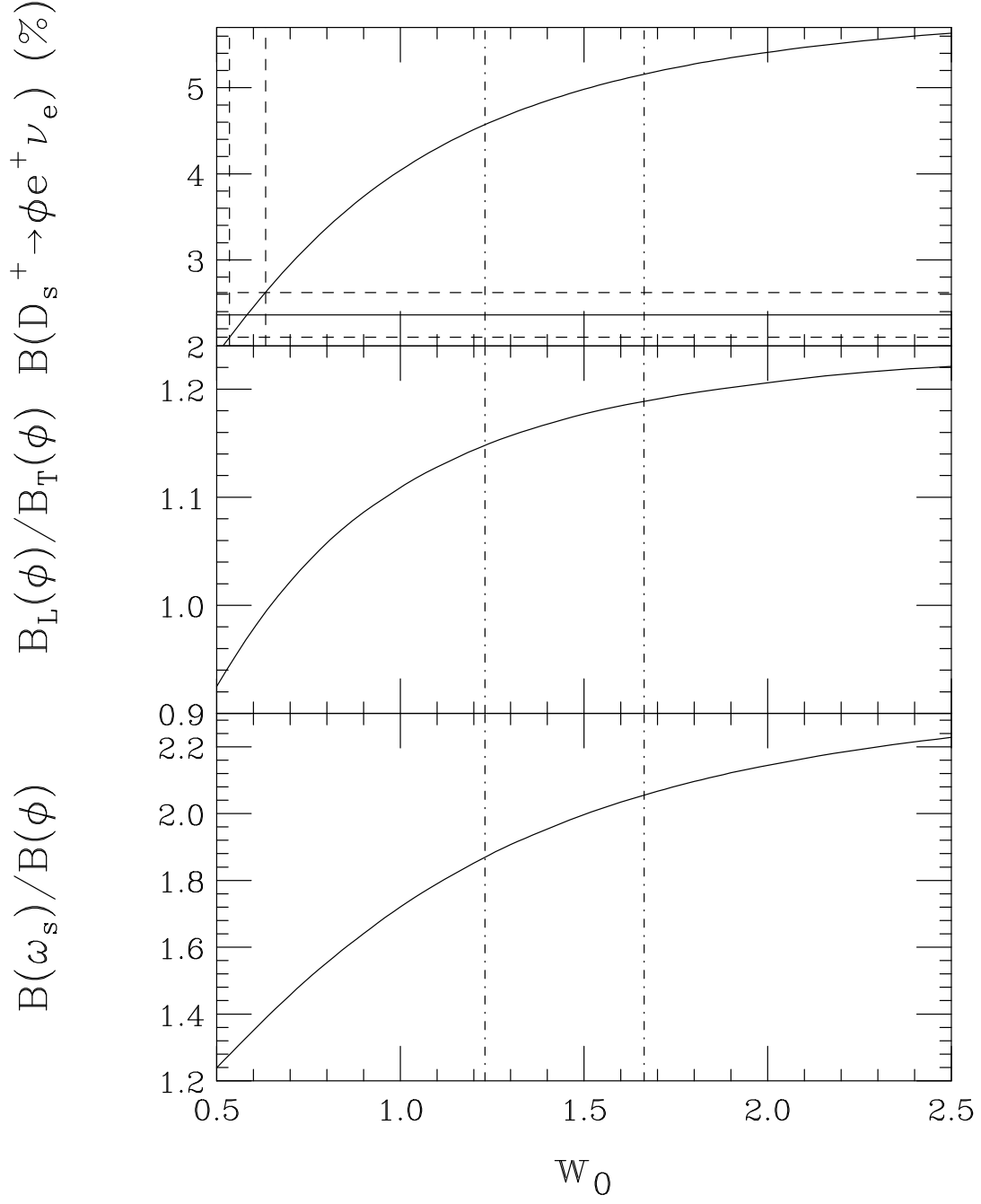
Figure 2: Dependence on form factor parameter w_0 of various predicted quantities. Vertical dash-dotted lines denote the limits on w_0 of the universal form factor considered in Ref. [13]. Top: $\mathcal{B}(D_s^+ \to \phi e^+ \nu_e)$; middle: ratio $\mathcal{B}_L(\phi)/\mathcal{B}_T(\phi)$; bottom: $\mathcal{B}(\omega_s)/\mathcal{B}(\phi)$, where ω_s denotes a pure $s\bar{s}$ state with the mass of ω. In the top figure, the solid and dashed horizontal lines correspond to the central and $\pm 1\sigma$ experimental values [2], while the dashed vertical lines represent the corresponding $\pm 1\sigma$ limits on w_0.

Table I: Measurements of the ratio of longitudinal to transverse ϕ polarization in $D_s^+ \to \phi\ell^+\nu_\ell$.

Reference	ℓ	Events	Ratio
E653 [15]	μ	19	$0.54 \pm 0.21 \pm 0.10$
E687 [16]	μ	90	$1.0 \pm 0.5 \pm 0.1$
CLEO [17]	e	308	$1.0 \pm 0.3 \pm 0.2$
Average [2]			0.72 ± 0.18

While small, this branching ratio could be detectable in the present CLEO sample [1] if backgrounds could be suitably suppressed and if δ were not anomalously small.

For completeness we discuss the decays $D_s \to (\eta, \eta')\ell\nu$. In principle these should be described by the same universal form factor as $D_s \to \phi\ell\nu$, with w^2 in Eq. (3) now defined as

$$w^2 = \frac{q^2 - q_{\max}^2}{M_{D_s} m_P} = \frac{q^2 - (M_{D_s} - m_P)^2}{M_{D_s} m_P} \ , \tag{9}$$

where P denotes the pseudoscalar meson (η or η'). The rather light mass of the η makes this approximation rather crude. The assumption of a universal pole in w^2 is not compatible with a universal pole in q^2, as one sees from the definition of w^2.

The observed branching ratios for D_s semileptonic decays involving η and η' are [2, 21]

$$\mathcal{B}(D_s \to \eta\ell\nu) = (2.9 \pm 0.6)\% \ , \quad \mathcal{B}(D_s \to \eta'\ell\nu) = (1.02 \pm 0.33)\% \ . \tag{10}$$

Charm nonleptonic decays [22, 23] and many other processes involving η and η' are well-approximated by the mixing scheme

$$\eta \simeq \frac{1}{\sqrt{3}}(s\bar{s} - u\bar{u} - d\bar{d}) \ , \quad \eta' \simeq \frac{1}{\sqrt{6}}(2s\bar{s} + u\bar{u} + d\bar{d}) \ . \tag{11}$$

With this scheme, the predicted branching ratios are plotted as functions of w_0 in Fig. 3.

A successful fit to $\mathcal{B}(D_s \to \eta\ell\nu)$ at the 1σ level requires $w_0 > 2.1$, while a successful fit to $\mathcal{B}(D_s \to \eta'\ell\nu)$ at the 1σ level requires $w_0 < 1.5$. This situation could be somewhat improved if the mixing scheme (11) were altered so that the strange quark admixture in the η were increased while the strange quark admixture in the η' were decreased. The scheme (11) corresponds to an octet-single mixing angle of $\theta = -\sin^{-1}(1/3) = -19.5°$. For the ISGW2 set of form factors [24] considered in Ref. [21], $\theta = -20°$ leads to the prediction $\mathcal{B}(D_s^+ \to \eta'e^+\nu_e)/\mathcal{B}(D_s^+ \to \eta e^+\nu_e) = 0.86$, to be compared with the measured value of $0.35 \pm 0.09 \pm 0.07$. Better agreement with the data is obtained for $\theta = -10°$, predicting this ratio to be 0.43. This is very close to the angle proposed by Isgur [25], $\theta = -9.74°$, in which

$$\eta \simeq \frac{1}{\sqrt{2}}s\bar{s} - \frac{1}{2}(u\bar{u} + d\bar{d}) \ , \quad \eta' \simeq \frac{1}{\sqrt{2}}s\bar{s} + \frac{1}{2}(u\bar{u} + d\bar{d}) \ . \tag{12}$$

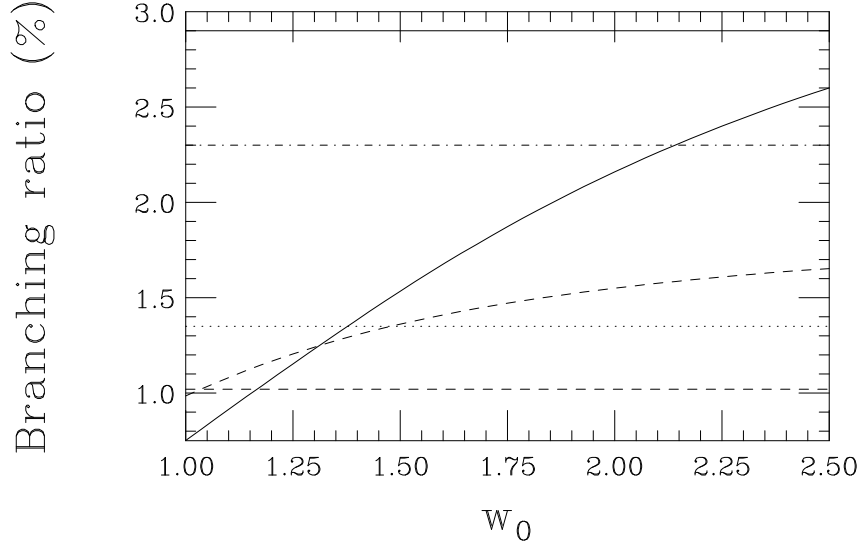The η'/η ratio in D_s semileptonic decays for the scheme (12) is half that for (11).

Figure 3: Predicted branching ratios for $D_s \to \eta \ell \nu$ (solid curve) and $(D_s \to \eta' \ell \nu$ (dashed curve) as a function of form factor parameter w_0, with η and η' assigned the quark content (11). Horizontal solid and dashed lines denote central values for $\mathcal{B}(D_s \to \eta \ell \nu)$ and $\mathcal{B}(D_s \to \eta' \ell \nu)$; horizontal dotdashed and dotted lines denote, respectively, -1σ and $+1\sigma$ experimental limits for $\mathcal{B}(D_s \to \eta \ell \nu)$ and $\mathcal{B}(D_s \to \eta' \ell \nu)$.

For the assignment (11), values of w_0 in the higher end of the range 1.23–1.66 considered earlier seem to represent an acceptable compromise. For $w_0 = 1.5$, the predicted η'/η ratio is 0.89 for this scheme, while for the assignment (12), one predicts $\mathcal{B}(D_s^+ \to \eta e^+ \nu_e) = 2.30\%$, $\mathcal{B}(D_s^+ \to \eta' e^+ \nu_e) = 1.02$, with an η'/η ratio of 0.44. Values of $1.5 \leq w_0 \leq 2.18$ give acceptable fits to both $\mathcal{B}(D_s^+ \to \eta e^+ \nu_e)$ and $\mathcal{B}(D_s^+ \to \eta' e^+ \nu_e)$ for the assignment (12), as illustrated in Fig. 4.

The spectra in $y = q^2/M_{D_s}^2$ are compared for $D_s \to \eta \ell \nu$ and $D_s \to \eta' \ell \nu$ in Fig. 5 for $w_0 = 1.5$. The enhancement of the spectrum for η' near $y = 0$ with respect to that for η represents the (lesser, greater) recoil of the (η', η) (and hence reflects a key aspect of the heavy-quark theory), but may be exaggerated by the considerable splitting of the η and η'.

III. RELATED HADRONIC PROCESSES

We now consider $D_s^+ \to \phi \pi^+$ in the heavy quark limit, again following Ref. [12]. The decay rate is predicted to be

$$\Gamma(D_s^+ \to \phi \pi^+) = \frac{[G_F V_{cs} V_{ud} f_\pi \xi(w_\pi^2)(1 + \sqrt{\zeta})]^2}{128\pi\sqrt{\zeta}} M_{D_s}^3 \lambda^{3/2}(1, \zeta, y_\pi) , \tag{13}$$

$$w_\pi^2 \equiv \frac{m_\pi^2 - (M_{D_s} - m_\phi)^2}{M_{D_s} m_\phi} = -0.439 , \quad y_\pi \equiv m_\pi^2/M_{D_s}^2 = 5.03 \times 10^{-3} . \tag{14}$$
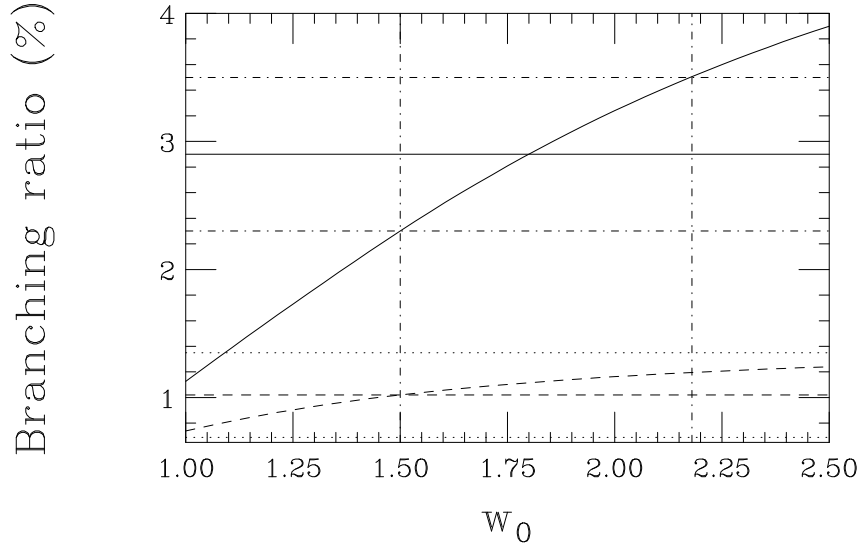
8

Figure 4: Same as Fig. 3 except that quark assignment (12) is used instead of (11). Here horizontal dotdashed and dotted lines denote, respectively, $\pm 1\sigma$ and $\pm 1\sigma$ experimental limits for $\mathcal{B}(D_s \to \eta\ell\nu)$ and $\mathcal{B}(D_s \to \eta'\ell\nu)$. Vertical dotdashed lines denote the limits $1.5 \leq w_0 \leq 2.18$ giving acceptable fits to both branching ratios.

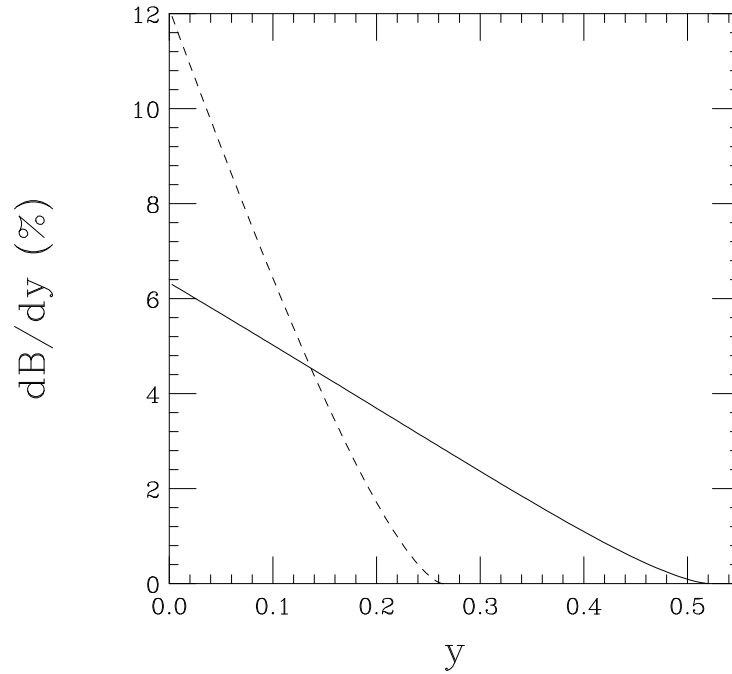Figure 5: Differential branching ratios (in percent) for $D_s \to \eta\ell\nu$ (solid curve) and $D_s \to \eta'\ell\nu$ (dashed curve) for $w_0 = 1.5$. Here the assignment (11) has been used. For the assignment (12), multiply the η curve by $3/2$ and the η' curve by $3/4$.
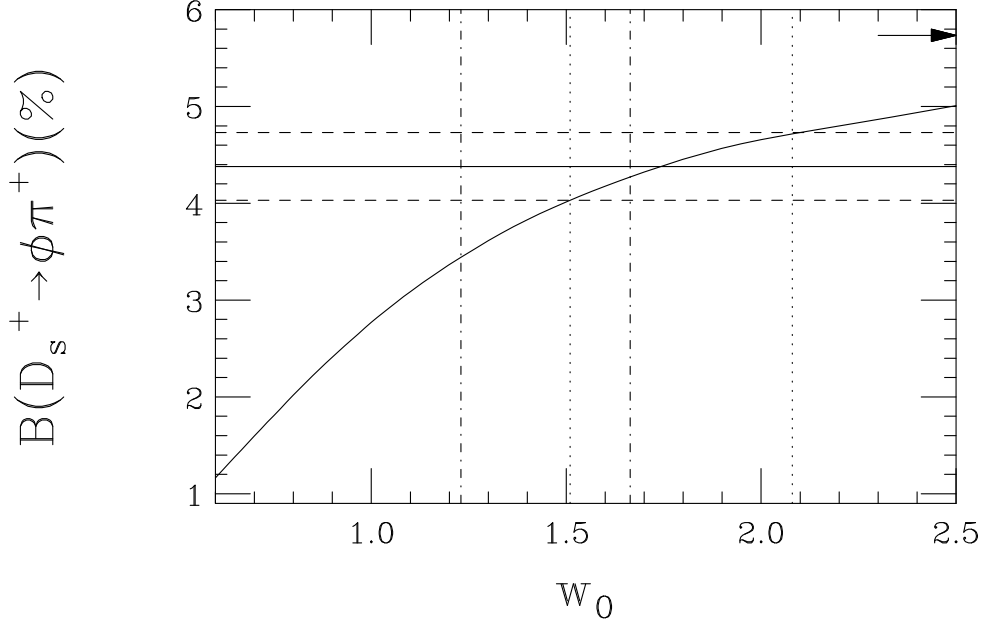
9

Figure 6: Branching ratio for $D_s^+ \to \phi\pi^+$ as a function of universal form factor parameter w_0. Horizontal solid and dashed lines denote central and $\pm 1\sigma$ experimental values [2]. Vertical dash-dotted lines denote limits associated with universal monopole form factor discussed in Ref. [13], while vertical dotted lines denote limits on w_0 based on $\pm 1\sigma$ experimental values. Arrow at upper right denotes predicted branching ratio for $w_0 \to \infty$.

Using $V_{cs} = V_{ud} = 0.974$, $f_\pi = 130.4$ MeV [26], we find

$$\mathcal{B}(D_s^+ \to \phi\pi^+) = \frac{5.73\%}{(1 - w_\pi^2/w_0^2)^2} \; . \tag{15}$$

We plot this quantity as a function of w_0 in Fig. 6.

As the experimental branching ratio is [2]

$$\mathcal{B}(D_s^+ \to \phi\pi^+) = (4.38 \pm 0.35)\% \; , \tag{16}$$

only a modest form factor suppression can be tolerated, whereas the value of $w_0 \simeq 0.6$ leading to an acceptable branching ratio for $D_s^+ \to \phi e^+ \nu_e$ implies $\mathcal{B}(D_s^+ \to \phi\pi^+) = 1.2\%$. We are thus led to consider the conservative limits $0.5 \leq w_0 \leq \infty$ in obtaining the range $1.2 \leq R_s \leq 2.4$ mentioned above. If we were to allow a fit to $\mathcal{B}(D_s^+ \to \phi\pi^+)$ at the $\pm 1\sigma$ level while demanding better agreement with other decays, we could demand $w_0 < 2.1$ (see Fig. 6). This would only reduce the upper limit on R_s by about 10%.

It has been argued that the K^+K^- S-wave contribution in $D_s^+ \to K^+K^-\pi^+$ cannot be overlooked [27], with

$$\frac{\Gamma(D_s^+ \to f_0(980)\pi^+ \to K^+K^-\pi^+)}{\Gamma(D_s^+ \to \phi\pi^+ \to K^+K^-\pi^+)} = 0.3 \pm 0.1 \; . \tag{17}$$

Applying this correction to the branching ratio (16), one obtains $\mathcal{B}(D_s^+ \to \phi\pi^+) = (4.38 \pm 0.35)\%/(1.3 \pm 0.1) = (3.37 \pm 0.37)\%$, implying $1.07 \leq w_0 \leq 1.36$, still within our range of consideration.

The hadronic process $D_s^+ \to \omega\pi^+$ would be related to $D_s^+ \to \omega e^+\nu_e$ if the only contributing amplitude were the color-favored subprocess $c \to s\pi^+$ followed by the mixing transition $s\bar{s} \to (u\bar{u} + d\bar{d})/\sqrt{2}$ giving an ω in the final state. This is not the case, however. A factorization calculation based on this assumption would predict

$$\frac{\mathcal{B}(D_s^+ \to \omega\pi^+)}{\mathcal{B}(D_s^+ \to \phi\pi^+)} = \left(\frac{p_{\pi\omega}^*}{p_{\pi\phi}^*}\right)^3 \tan^2\delta \ , \tag{18}$$

where $p_{\pi\omega}^* = 822$ MeV/c and $p_{\pi\phi}^* = 712$ MeV/c are center-of-mass 3-momenta for the respective decays. With $\mathcal{B}(D_s^+ \to \phi\pi^+) = (4.38 \pm 0.35)\%$, this implies $\mathcal{B}(D_s^+ \to \omega\pi^+) = (2.3 \pm 0.2) \times 10^{-4}(\delta/3.34°)^2$. The experimental value is considerably larger [2, 28],

$$\mathcal{B}(D_s^+ \to \omega\pi^+) = (2.5 \pm 0.9) \times 10^{-3} \ , \tag{19}$$

implying the importance of a weak annihilation contribution [23].

As has been mentioned, the $c\bar{s} \to u\bar{d}$ "annihilation" amplitude A, if interpreted literally, would be subject to helicity suppression, so in flavor SU(3) treatments [22, 23] it must represent a shorthand for rescattering contributions. Further evidence for this viewpoint comes from the observation of the decay $D_s^+ \to p\bar{n}$ [29]. If interpreted literally in terms of the production of $p\bar{n}$ by the weak current from $c\bar{s}$ annihilation (i.e., if treated by a factorization hypothesis), this process would be highly suppressed by PCAC [30], whereas the observed branching ratio is $\mathcal{B}(D_s^+ \to p\bar{n}) = (1.30 \pm 0.36^{+0.12}_{-0.16}) \times 10^{-3}$ [29].

The decays $D_s^+ \to \eta\pi^+$ and $D_s^+ \to \eta'\pi^+$ may be related to $D_s^+ \to \phi\pi^+$ in the heavy-quark limit. (For a study of $D_s^+ \to \eta\pi^+$ and $D_s^+ \to \eta'\pi^+$ using factorization of the tree amplitude, see Ref. [31].) In the treatment of Ref. [12], the ratio of partial widths contributed by the factorized tree ("T") amplitude is given in the limit of degenerate $s\bar{s}$ vector V_s and pseudoscalar P_s masses by

$$\frac{\Gamma(D_s^+ \to V_s\pi^+)_T}{\Gamma(D_s^+ \to P_s\pi^+)_T} = \left[\frac{1 + \sqrt{\zeta}}{1 - \sqrt{\zeta}}\right]^2 \frac{\lambda(1, \zeta, y_\pi)}{[(1 + \sqrt{\zeta})^2 - y_\pi^2]^2} \ , \tag{20}$$

where $\sqrt{\zeta} \equiv M_{V,P}/M_{D_s}$ and $y_\pi \equiv m_\pi^2/M_{D_s}^2$. Neglecting the small quantity y_π, we find in this limit that the right-hand side reduces to unity, so

$$\Gamma(D_s \to V_s\pi^+)_T = \Gamma(D_s \to P_s\pi^+)_T \ , \tag{21}$$

or, independently of the precise nature of octet-singlet mixing in η and η', and neglecting phase space differences,

$$\mathcal{B}(D_s^+ \to \phi\pi^+)_T = \mathcal{B}(D_s^+ \to \eta\pi^+)_T + \mathcal{B}(D_s^+ \to \eta'\pi^+)_T \ . \tag{22}$$

The decay $D_s^+ \to \phi\pi^+$ is expected to be dominated by the T amplitude [23], while small corrections to T dominance are due to the annihilation amplitude A in $D_s^+ \to (\eta, \eta')\pi^+$ [22]. The branching ratios for $D_s^+ \to (\eta, \eta')\pi^+$ are [2]

$$\mathcal{B}(D_s^+ \to \eta\pi^+) = (1.58 \pm 0.21)\% \ , \quad \mathcal{B}(D_s^+ \to \eta'\pi^+) = (3.8 \pm 0.4)\% \ , \tag{23}$$

while the contributions of the tree amplitudes to these decay widths are [22]

$$
\begin{aligned}
\mathcal{B}(D_s^+ \to \eta\pi^+)_T &= \left(\frac{1.605}{1.50}\right)^2 (1.58 \pm 0.21)\% = (1.81 \pm 0.24)\% , \\
\mathcal{B}(D_s^+ \to \eta'\pi^+)_T &= \left(\frac{2.27}{2.55}\right)^2 (3.8 \pm 0.4)\% = (3.01 \pm 0.32)\%.
\end{aligned}
\tag{24}
$$

The sum rule (22) then reads

$$
(4.38 \pm 0.35)\% = (4.82 \pm 0.40)\% ,
\tag{25}
$$

which is satisfactorily obeyed. A similar confirmation of the heavy-quark relation between tree amplitudes in PP and VP decays of charmed mesons was obtained in Refs. [22] and [23] by comparing their contributions in $D \to \overline{K}\pi$ and $D \to \overline{K}^*\pi$ decays. (See, in particular, Eqs. (18) and (19) in Ref. [23].)

IV. WEAK ANNIHILATION IN $D_s^+ \to \omega\pi^+$ and $D_s^+ \to \omega e^+\nu_e$.

A difficulty (see, e.g., Refs. [23, 32, 33]) in ascribing the decay $D_s^+ \to \omega\pi^+$ to the weak subprocess $c\bar{s} \to u\bar{d}$ is that because the final $u\bar{d}$ state has odd G-parity (as does a pion), it cannot decay to $\omega\pi^+$, which has even G-parity [7, 34, 35]. In a flavor-symmetric description [32], the decays $D_s^+ \to \rho^0\pi^+$ and $D_s^+ \to \omega\pi^+$ both involve amplitudes A_V and A_P, where the subscript denotes whether the $\bar{d}$ quark in the $c\bar{s} \to u\bar{d}$ subprocess is included in a pseudoscalar (P) or a vector (V) meson. These are required to cancel one another for $D_s^+ \to \omega\pi^+$ in order to enforce the G-parity selection rule; they will then add in $D_s^+ \to \rho^0\pi^+$. However, one sees a branching ratio $\mathcal{B}(D_s^+ \to \omega\pi^+) = (2.5 \pm 0.9) \times 10^{-3}$, while $D_s^+ \to \rho^0\pi^+$ is only quoted as "not seen" [2].

Moreover, annihilation topologies, if interpreted literally in terms of quarks, are subject to helicity selection rules leading to their suppression, so one must interpret them as encoding the effects of rescattering. The authors of Ref. [34] ascribe the decay $D_s^+ \to \omega\pi^+$ to the weak decay $D_s^+ \to \overline{K}^{(*)0}K^{(*)+}$ followed by $\overline{K}^{(*)0}K^{(*)+} \to \omega\pi^+$. A successful prediction of the branching ratio for $D_s^+ \to \omega\pi^+$ was made on the basis of final-state interactions in Ref. [36]. However, within these two frameworks there is no corresponding process contributing to $D_s^+ \to \omega\ell^+\nu$.

An alternate possibility is that the decay $D_s^+ \to \omega\pi^+$ proceeds through pre-radiation of the ω, whether via violation of the OZI rule or rescattering. An example of the latter mechanism would be the dissociation of the D_s^+ into two-meson states such as $D^{(*)0}K^{(*)+}$ and $D^{(*)+}K^{(*)0}$. The two mesons can be PV, VP, or VV and must be in a relative P-wave; PP is forbidden by parity. The two mesons then rescatter strongly to $(c\bar{s})\omega$ and the virtual $c\bar{s}$ state decays weakly to π^+.

A correponding mechanism can generate the decay $D_s^+ \to \omega e^+\nu_e$. Here, the virtual $c\bar{s}$ (which can now be spin-1, and hence not subject to helicity suppression) decays to a lepton pair. We may estimate very crudely the branching ratio for this process if the corresponding process (described above) is responsible for $D_s^+ \to \omega\pi^+$. Neglecting all kinematic factors, we expect

$$
\frac{\mathcal{B}(D_s \to \omega\ell\nu)}{\mathcal{B}(D_s \to \phi\ell\nu)} = \frac{\mathcal{B}(D_s^+ \to \omega\pi^+)}{\mathcal{B}(D_s^+ \to \phi\pi^+)} .
\tag{26}
$$

Using the branching ratios quoted earlier, we infer

$$\mathcal{B}(D_s \rightarrow \omega\ell\nu)_{\mathrm{WA}} = (1.3 \pm 0.5) \times 10^{-3} \,, \tag{27}$$

roughly an order of magnitude larger than one would conclude if ω–ϕ mixing were solely responsible for the decay.

We have neglected differences in form factor behavior which are to be expected for the WA process, since it is expected to be peaked at maximum q^2. This peaking occurs both in the scenario where the ω is emitted via an OZI-suppressed three-gluon coupling from the initial $c\bar{s}$ system, and where rescattering gives rise to a virtual D_s^* which then decays to $\ell\nu$. In the latter case, high q^2 is favored by proximity to the D_s^* pole. By contrast, as can be seen in Fig. 1, one does not expect peaking for $D_s \rightarrow \phi\ell\nu$ at high q^2 except for the lowest values of w_0. The peaking of the spectrum for $D_s \rightarrow \omega\ell\nu$ at maximum q^2 will be one of the hallmarks of the WA process.

V. CONCLUSION

We have considered the ratio $R = \mathcal{B}(D_s^+ \rightarrow \omega e^+\nu_e)/\mathcal{B}(D_s^+ \rightarrow \phi e^+\nu_e)$ as a test of ϕ–ω mixing in the absence of nonperturbative enhancements, and, in the event that the ratio exceeds a nominal estimate, as possible evidence for such enhancements, termed "weak annihilation" [4]. We find for ϕ–ω mixing a range

$$R = (4.1 - 8.2)(\delta/3.34°)^2 \times 10^{-3} \,, \tag{28}$$

where δ is the ω–ϕ mixing angle. The value $\delta = 3.34°$ is obtained in one mass-independent analysis [9], while a considerably smaller value of $-0.45°$ at m_ω is found when the angle is allowed to vary with mass [10].

Given the experimental branching ratio $\mathcal{B}(D_s^+ \rightarrow \phi e^+\nu_e) = (2.36 \pm 0.26)\%$, we conclude that any value of $\mathcal{B}(D_s^+ \rightarrow \omega e^+\nu_e)$ exceeding $(8.2 \times 10^{-3}) \cdot (2.6\%) \simeq 2 \times 10^{-4}$ is unlikely to be explainable via ω–ϕ mixing, and would provide evidence for nonperturbative effects such as those discussed in Refs. [4]. A crude estimate based on comparing hadronic and semileptonic processes gives a branching ratio $\mathcal{B}(D_s^+ \rightarrow \omega e^+\nu_e) = (1.3 \pm 0.5) \times 10^{-3}$, nearly an order of magnitude higher than the values from ω–ϕ mixing alone.

ACKNOWLEDGMENTS

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